Paragraph -1

EXERCISE

1. Fill in the blanks:
   (a) 1 lakh = ________ ten thousand.
   (b) 1 million = ________ hundred thousand.
   (c) 1 crore = ________ ten lakh.
   (d) 1 crore = ________ million.
   (e) 1 million = ________ lakh.

2. Place commas correctly and write the numerals:
   (a) Seventy three lakh seventy five thousand three hundred seven.
   (b) Nine crore five lakh forty one.
   (c) Seven crore fifty two lakh twenty one thousand three hundred two.
   (d) Fifty eight million four hundred twenty three thousand two hundred two.
   (e) Twenty three lakh thirty thousand ten.

3. Insert commas suitably and write the names according to Indian System of Numeration :
   (a) 87595762 (b) 8546283 (c) 99900046 (d) 98432701

4. Insert commas suitably and write the names according to International System of Numeration :
   (a) 78921092 (b) 7452283 (c) 99985102 (d) 48049831

Large Numbers in Practice

In earlier classes, we have learnt that we use centimetre (cm) as a unit of length. For measuring the length of a pencil, the width of a book or notebooks etc., we use centimetres. Our ruler has marks on each centimetre. For measuring the thickness of a pencil, however, we find centimetre too big. We use millimetre (mm) to show the thickness of a pencil.

(a) 10 millimetres = 1 centimetre

To measure the length of the classroom or the school building, we shall find centimetre too small. We use metre for the purpose.

(b) 1 metre = 100 centimetres

= 1000 millimetres

Even metre is too small, when we have to state distances between cities, say, Delhi and Mumbai, or Chennai and Kolkata. For this we need kilometres (km).

Try These

1. How many centimetres make a kilometre?
2. Name five large cities in India. Find their population. Also, find the distance in kilometres between each pair of these cities.

(c) 1 kilometre = 1000 metres

How many millimetres make 1 kilometre?

Since 1 m = 1000 mm

1 km = 1000 m = 1000 × 1000 mm = 10,00,000 mm

We go to the market to buy rice or wheat; we buy it in kilograms (kg). But items like ginger or chillies which we do not need in large quantities, we buy in grams (g). We know 1 kilogram = 1000 grams.

Have you noticed the weight of the medicine tablets given to the sick? It is very small. It is in milligrams (mg).

1 gram = 1000 milligrams.

What is the capacity of a bucket for holding water? It is usually 20 litres (ℓ). Capacity is given in litres. But sometimes we need a smaller unit, the millilitres. A bottle of hair oil, a cleaning liquid or a soft drink have labels which give the quantity of liquid inside in millilitres (ml).

1 litre = 1000 millilitres.

Note that in all these units we have some words common like kilo, milli and centi. You should remember that among these **kilo** is the greatest and **milli** is the smallest; kilo shows 1000 times greater, milli shows 1000 times smaller, i.e. 1 kilogram = 1000 grams, 1 gram = 1000 milligrams.

Try These

1. How many milligrams make one kilogram?
2. A box contains 2,00,000 medicine tablets each weighing 20 mg. What is the total weight of all the tablets in the box in grams and in kilograms?

Similarly, centi shows 100 times smaller, i.e. 1 metre = 100 centimetres.

Paragraph -2

Tests for Divisibility of Numbers

Is the number 38 divisible by 2? by 4? by 5?

By actually dividing 38 by these numbers we find that it is divisible by 2 but not by 4 and by 5.

Let us see whether we can find a pattern that can tell us whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, 10 or 11. Do you think such patterns can be easily seen?

Divisibility by 10 : Charu was looking at the multiples of 10. The multiples are 10, 20, 30, 40, 50, 60, ... She found something common in these numbers. Can you tell what? Each of these numbers has 0 in the ones place.

She thought of some more numbers with 0 at ones place like 100, 1000, 3200, 7010. She also found that all such numbers are divisible by 10.

She finds that **if a number has 0 in the ones place then it is divisible by 10.**

Can you find out the divisibility rule for 100?

Divisibility by 5 : Mani found some interesting pattern in the numbers 5, 10, 15, 20, 25, 30, 35, ... Can you tell the pattern? Look at the units place. All these numbers have either 0 or 5 in their ones place. We know that these numbers are divisible by 5.

Mani took up some more numbers that are divisible by 5, like 105, 215, 6205, 3500. Again these numbers have either 0 or 5 in their ones places.

He tried to divide the numbers 23, 56, 97 by 5. Will he be able to do that? Check it. He observes that **a number which has either 0 or 5 in its ones place is divisible by 5,** other numbers leave a remainder.

Is 1750125 divisible 5?

Divisibility by 2 : Charu observes a few multiples of 2 to be 10, 12, 14, 16... and also numbers like 2410, 4356, 1358, 2972, 5974. She finds some pattern in the ones place of these numbers. Can you tell that? These numbers have only the digits 0, 2, 4, 6, 8 in the ones place.

She divides these numbers by 2 and gets remainder 0.

She also finds that the numbers 2467, 4829 are not divisible by 2. These numbers do not have 0, 2, 4, 6 or 8 in their ones place.

Looking at these observations she concludes that **a number is divisible by 2 if it has any of the digits 0, 2, 4, 6 or 8 in its ones place.**

Divisibility by 3 : Are the numbers 21, 27, 36, 54, 219 divisible by 3? Yes, they are.

Are the numbers 25, 37, 260 divisible by 3? No.

Can you see any pattern in the ones place? We cannot, because numbers with the same digit in the ones places can be divisible by 3, like 27, or may not be divisible by 3 like 17, 37. Let us now try to add the digits of 21, 36, 54 and 219. Do you observe anything special ? $2+1=3$, $3+6=9$, $5+4=9$, $2+1+9=12$. All these additions are divisible by 3.

Add the digits in 25, 37, 260. We get $2+5=7$, $3+7=10$, $2+6+0 = 8$.

These are not divisible by 3.

We say that **if the sum of the digits is a multiple of 3, then the number is divisible by 3.**

Is 7221 divisible by 3?

Divisibility by 6 : Can you identify a number which is divisible by both 2 and 3? One such number is 18. Will 18 be divisible by $2\times3=6$? Yes, it is.

Find some more numbers like 18 and check if they are divisible by 6 also.

Can you quickly think of a number which is divisible by 2 but not by 3?

Now for a number divisible by 3 but not by 2, one example is 27. Is 27 divisible by 6? No. Try to find numbers like 27.

From these observations we conclude that **if a number is divisible by 2 and 3 both then it is divisible by 6 also.**

Paragraph -3

1. The distance between the end points of a line segment is its *length*.
2. A graduated *ruler* and the *divider* are useful to compare lengths of line segments.
3. When a hand of a clock moves from one position to another position we have an example for an *angle*.

   One full turn of the hand is 1 *revolution*.

   A *right angle* is ¼ revolution and a *straight angle* is ½ a revolution .

   We use a *protractor* to measure the size of an angle in degrees.

   The measure of a right angle is 90° and hence that of a straight angle is 180°.

   An angle is *acute* if its measure is smaller than that of a right angle and is *obtuse* if its measure is greater than that of a right angle and less than a straight angle.

   A *reflex angle* is larger than a straight angle.

4. Two intersecting lines are *perpendicular* if the angle between them is 90°.
5. The *perpendicular bisector* of a line segment is a perpendicular to the line segment that divides it into two equal parts.
6. Triangles can be classified as follows based on their angles:

Nature of angles in the triangle	*Name*
Each angle is acute	Acute angled triangle
One angle is a right angle	Right angled triangle
One angle is obtuse	Obtuse angled triangle

7. Triangles can be classified as follows based on the lengths of their sides:

Nature of sides in the triangle	*Name*
All the three sides are of unequal length	Scalene triangle
Any two of the sides are of equal length	Isosceles triangle
All the three sides are of equal length	Equilateral triangle

8. Polygons are named based on their sides.

Number of sides	*Name of the Polygon*
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
8	Octagon

9. Quadrilaterals are further classified with reference to their properties.

Properties	*Name of the Quadrilateral*
One pair of parallel sides	Trapezium
Two pairs of parallel sides	Parallelogram
Parallelogram with 4 right angles	Rectangle
Parallelogram with 4 sides of equal length	Rhombus
A rhombus with 4 right angles	Square

10. We see around us many *three dimensional shapes*. Cubes, cuboids, spheres, cylinders, cones, prisms and pyramids are some of them.

Paragraph -4

Profit-Loss

- **Let us remember :**
  - The amount at which a trader buys a product is known as its **Cost price.**
  - After buying a product, the additional expenditure incurred on a product is called **Additional expense.** The expenditure on labour, rent, octroi tax, maintenance work is called additional expense.
  - **The sum of cost price and additional expense is called Net price.** When there is no extra expenditure, the cost price is considered the Net price.

    Net price = Cost price + Additional expense
  - The amount at which a trader sells the product is called the **Sale price.**
  - The excess amount got over the cost price after selling the product is called the **profit.**

    Profit = Sale price − Net price

    So, **Profit = S.P. − N.P.**
  - The amount got at the time of sale less than the net amount is called the **Loss.**

    Loss = Net price − Sale price

    So, **Loss = N.P. − S.P.**

    When there is profit, **Sale price = Net price + Profit**

    When there is loss, **Sale price = Net price − Loss**

Now, find the answers from above information :

A trader bought a TV for ₹ 9950. He paid ₹ 50 as labour charge to bring the TV home. Selling the TV at ₹ 10,700, he earned the profit of ₹ 700.

Cost price	= ₹	Additional expense	= ₹ ....
Net price	= ₹	Sale Price	= ₹ ....
Profit	= ₹	Profit	= .... %

- **Let's learn something new :**

  - In Std. 6, we learnt how to find profit or loss in terms of percentage. Now on the basis of profit and loss percentage and net price, let's understand how to find the selling price.

Example 1 : To earn a profit of 10 %, at what price should a product costing ₹ 400 be sold ?

Solution : Method - 1

Profit on the cost price of ₹ 100 = ₹ 10

∴ Profit on the cost price of ₹ 400 = $\left(10\times\frac{400}{100}\right)$

= ₹ 40

∴ Sale price = Profit + Net price

= ₹ (40 + 400)

= ₹ 440

Method - 2

To earn a profit of 10 %, a product of ₹ 100 should be sold at ₹ 110.

The sale price of a product of

₹ 100 = ₹ 110

Then, the sale price of a product of

₹ 400 = ₹ $\frac{110\times 400}{100}$ = ₹ 440

∴ To earn a profit of 10 % on a product of ₹ 400, it should be sold at ₹ 440.

1. Fill the following Table after calculation :

Sr. No.	Cost price (in ₹)	Addi. Exp. (in ₹)	Profit (in %)	Loss (in %)	Sale price
1.	60	–	5	–	....
2.	40	–	10	–	....
3.	1000	–	12	–	....
4.	240	–	–	15	....
5.	1500	–	–	5	....
6.	24	–	–	12.5	....
7.	1650	150	–	5	....
8.	750	50	–	10.5	....
9.	3800	200	15.5	–	....

Paragraph -5

- **Properties for addition of rational numbers :**

Note the result by adding the following rational numbers :

No.	Addition	Result	Properties
(1)	$\frac{4}{7}+\left(-\frac{2}{3}\right)=...$ $\left(-\frac{1}{4}\right)+\frac{3}{8}=...$	Is resulting number a rational number ?	**Closure property :** The addition of any two rational numbers is a rational number.
(2)	$\frac{4}{7}+\left(-\frac{2}{3}\right)=...$ $\left(-\frac{2}{3}\right)+\frac{4}{7}=...$	How result is obtained when order is changed.	**Commutative property :** Two rational numbers can be added in any order but the result is same.
(3)	$\left[\left(-\frac{3}{4}\right)+\frac{1}{2}\right]+\frac{2}{6}=...$ $\left(-\frac{3}{4}\right)+\left[\frac{1}{2}+\frac{2}{6}\right]=...$	How result is obtained when group is changed ?	**Associative property :** For any three rational numbers if in the group of any two numbers, third number is added, the result is same.
(4)	$\left(-\frac{3}{4}\right)+0=...$ $0+\frac{2}{3}=...$	How result is obtained when addition is done with zero (0) ?	**Existence of identity element :** For addition of a rational number and zero, we get the same rational number. Therefore, zero (0) is the identity element for addition.
(5)	$\left(-\frac{7}{17}\right)+\frac{7}{17}=...$ $\frac{3}{5}+\left(-\frac{3}{5}\right)=...$	What is the result when two opposite rational numbers are added ?	For any rational number there always exist an opposite number such that addition of both number is zero.

- **Properties for multiplication of rational numbers :**

Note the result by multiplication of following rational numbers :

No.	Addition	Result	Properties
(1)	$0 \times \frac{5}{9} = ...$ $\left(-\frac{3}{2}\right) \times \frac{2}{6} = ...$	Is resulting number a rational number ?	**Closure property :** The addition of any two rational numbers is a rational number.
(2)	$\left(-\frac{2}{5}\right) \times \frac{10}{3} = ...$ $\frac{10}{3} \times \left(-\frac{2}{5}\right) = ...$	How result is obtained when order of numbers are changed.	**Commutative property :** When two rational numbers are multiplied in any order then the result is same.
(3)	$\left[\left(-\frac{1}{3}\right)\times\frac{3}{4}\right]\times\frac{6}{7} = ...$ $\left(-\frac{1}{3}\right)\times\left[\frac{3}{4}\times\frac{6}{7}\right] = ...$	How result is obtained when group is changed ?	**Associative property :** For any three rational numbers if in the group of any two number, the third number is multiplied, the result is same.
(4)	$\left(-\frac{4}{9}\right) \times 1 = ...$ $\frac{3}{7} \times 1 = ...$	How result is obtained when any number is multiplied with one ?	**Existence of identity element :** Multiplication of any rational number and 1 is always the same rational number. Therefore, 1 is the identity element for multiplication.
(5)	$\frac{3}{5} \times \frac{5}{3} = ...$ $\left(-\frac{1}{2}\right) \times \left(-\frac{2}{1}\right) = ...$	What is the result when two inverse (reciprocal) rational numbers are multiplied ?	For any rational number there always exist a reciprocal number such that multiplication of both number is one.

Paragraph -6

- **Some special sets :**

Empty Set :

The set without any member is called the empty set. It is denoted by symbol ϕ (phi) or "{ }".

For example : A = {x / x is a prime number less than 2, $x \in$ N}, then A = ϕ or A = { }

B = {x / x is a prime of female chief minister of Gujarat}, then B = ϕ or B = { }

Singleton Set :

A set having only one element (member) is called the singleton set.

For example : P = {x / x is an odd prime number less than 5}

P = {3}

Finite Set :

If the number of members in a set is a definite non-negative integer, then the set is called a finite set. For example, A = {1, 2, 3, ... 10}

Here, 1 to 10 natural numbers are included in set A, which can be counted. Therefore, the number of members of set A is 10 which is definite. Therefore, the set A is a finite set.

The number of members of a set A is denoted by n(A).

Here, the number of members of set A is 10. Therefore, n(A) = 10

- The empty set is also a finite set.

Infinite Set :

A set which is not finite is an infinite set.

For example : A = {x/x is a natural number} $\therefore$ A = {1, 2, 3, 4, ...}

Here, there is no end of the list of members in set A. Such set is called an infinite set. To an infinite set after writing some members, generally three points are denoted.

- The set of natural numbers is denoted by a special symbol N.

  N = {1, 2, 3, ...}
- The set of whole numbers is denoted by a special symbol W.

  W = {0, 1, 2, 3, ...}
- The set of integers is denoted by a special symbol Z.

  Z = {..., −3, −2, −1, 0, 1, 2, 3, ...}
- The set of rational numbers (quotients) is denoted by a special symbol Q.

  $$Q = \left\{\frac{p}{q} \mid p \in Z, q \in N\right\}$$
- N, W, Z and Q, all these are infinite sets.
- Why is the set of leaves of a tree is called an infinite set ? Why ? Think !

- **Equal Set :**

If set A and B contain the same elements, then set A and set B are said to be equal sets. Symbolically it is written as A = B.

For example : A = $\{x / x$ is a natural number less than 5$\}$, A = {1, 2, 3, 4}

B = $\{x / x$ is a factor of 12 less than 5$\}$, B = {1, 2, 3, 4}

Here, set A and set B have the same elements, therefore set A and set B are equal sets.

$\therefore$ A = B

Additionally here A $\subset$ B and B $\subset$ A.

$\therefore$ A $\subset$ B and B $\subset$ A, then A = B

- **One to One Correspondence :**

Suppose there are 10 students in a class. A unique (one and only one) roll number is given to each student.

(1) Utsav or 1 $\leftrightarrow$ Utsav

(2) Vijay or 2 $\leftrightarrow$ Vijay

(3) Chahana or 3 $\leftrightarrow$ Chahana

⋮ ⋮ ⋮

(10) Rehana or 10 $\leftrightarrow$ Rehana

Therefore, there is only one number from 1 to 10 corresponding to each student, which is his roll number and no student have two roll numbers because there will not be two students with same roll number. Such correspondence is called one to one correspondence.

Now, A = {1, 2, 3} and B = $\{a, b, c\}$, then there can be 6 correspondence between them.

(i)	(ii)	(iii)	(iv)	(v)	(vi)
$1 \leftrightarrow a$	$1 \leftrightarrow a$	$1 \leftrightarrow b$	$1 \leftrightarrow b$	$1 \leftrightarrow c$	$1 \leftrightarrow c$
$2 \leftrightarrow b$	$2 \leftrightarrow c$	$2 \leftrightarrow a$	$2 \leftrightarrow c$	$2 \leftrightarrow a$	$2 \leftrightarrow b$
$3 \leftrightarrow c$	$3 \leftrightarrow b$	$3 \leftrightarrow c$	$3 \leftrightarrow a$	$3 \leftrightarrow b$	$3 \leftrightarrow a$

- **Equivalent Set :**

If the number of the elements of two finite sets is the same, then they are called equivalent sets. Its symbol is '~'.

$A = \{1, 4, 6\}$ $\quad$ $B = \{x, y, z\}$

$n(A) = 3$ $\quad$ $n(B) = 3$

Here, $n(A) = n(B)$. Therefore set A and set B are equivalent sets.

Symbolically they are written as $A \sim B$.

- **Universal Set :**

Generally, in the discussion about sets, all sets are assumed to be the subsets of a definite set. This definite set with respect to its subsets is called the universal set. The symbol U is used for universal set.

For example, with reference to the set of all students of the school, the set of the player of Kho-kho team, the set of players of Kabaddi team, the set of members of the prayer committee, the set of students of VIII Standard etc are the subsets of the set of the students of school. Therefore, in this reference the set of the students of the school is the universal set.

Paragraph -7

- **A closed figure made by four line-segments having four angles and not intersecting each other at any point except the end points is a quadrilateral.**

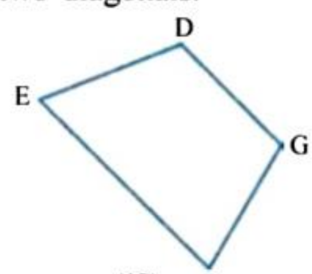
☐ ABCD can be written as in set form as under :

☐ ABCD = $\overline{AB} \cup \overline{BC} \cup \overline{CD} \cup \overline{DA}$

- Therefore, a quadrilateral is the union of four line-segments.
- Each line-segment which is formed by connecting opposite vertices of a quadrilateral is called a diagonal.
- If the diagonals of a quadrilateral intersect each other then that quadrilateral is called a convex quadrilateral.
- If the diagonals of a quadrilateral do not intersect each other then that quadrilateral is called a concave quadrilateral.
- Each quadrilateral contains four sides, four angles and two diagonals.

Here, we will discuss about the convex quadrilateral.

In the figure, D, E, F, G are the vertices of a quadrilateral, therefore, its name is given quadrilateral DEFG. Symbolically it is written as ☐ DEFG.

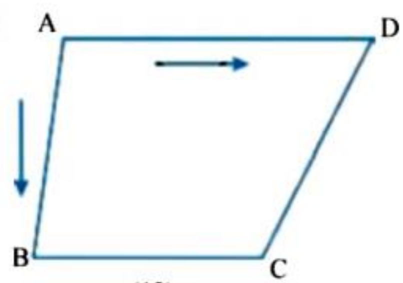


(12)

Read as : Quadrilateral DEFG

- **Naming of quadrilateral :**

☐ ABCD
☐ BCDA
☐ CDAB
☐ DABC



(13)

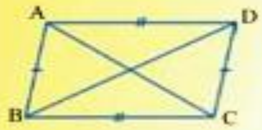
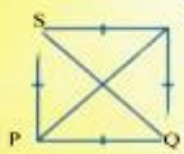
☐ ADCB
☐ DCBA
☐ CBAD
☐ BADC

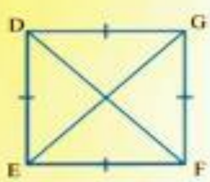
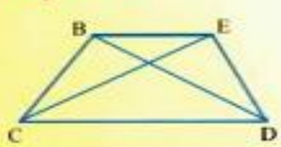
- A quadrilateral can be given name by two methods as clockwise and anticlockwise.
- Fill the details of the Table on the basis of above figures (12) and (13) :

Figures	Sides	Angles	Diagonals	Diagonals intersect ?
(12)				
(13)				

- **Types of quadrilateral :**

See and understand :

No.	Figure and name	Definition	Characteristics
1.	**Parallelogram** 	If both the pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is called a parallelogram. $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{CD}$	• Diagonals bisect each other. • Diagonals are not of equal measures. • The measures of opposite sides are equal. • The measures of opposite angles are equal.
2.	**Rhombus** 	If all the sides of a parallelogram are equal, then it is called a rhombus. $\overline{QR} \parallel \overline{PS}$ and $\overline{PQ} \parallel \overline{SR}$ $QR = RS = SP = PQ$	• Diagonals are not of equal measurement. • Diagonals bisect each other at right angles.

No.	Figure and Name	Definition	Characteristics
3.	**Rectangle** 	If all the angles of a parallelogram are right angles, then the parallelogram is called a rectangle. $\overline{LO} \parallel \overline{MN}$, $\overline{LM} \parallel \overline{ON}$ $m\angle L = m\angle M = m\angle N = m\angle O = 90^\circ$	• Diagonals are equal in measures. • Diagonals bisect each other.
4.	**Square** 	If the measures of four angles and four sides are equal, then that parallelogram is called a square. $\overline{DG} \parallel \overline{EF}$, $\overline{DE} \parallel \overline{GF}$ $DG = GF = FE = ED$ $m\angle D = m\angle E = m\angle F = m\angle G = 90^\circ$	• The measures of diagonals are equal. • Diagonals bisect each other at right angles.
5.	**Trapezium** 	If in a quadrilateral one and only one pair of opposite sides are parallel then such quadrilateral is called a trapezium. $\overline{BE} \parallel \overline{CD}$	• The measures of diagonals are not equal. • Diagonals do not bisect each other.

- **Remember :**
  - The name of a quadrilateral **can** be given by two ways : clockwise and anti-clockwise.
  - The quadrilateral **name can be written by starting** with any vertex.
  - Each quadrilateral **has two pairs of opposite** sides and two pairs of opposite angles.
  - Each quadrilateral **has four pairs of** adjacent sides and four pairs of adjacent angles.
  - The sum of the measures of all our angles of any quadrilateral is 360°.

  Therefore, for $\square ABCD$, $m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$

Paragraph -8

Plotting a Point in the Plane if its Coordinates are Given

Let us obtain a point in the plane corresponding to the ordered pair (2, 3). The x-coordinate and y-coordinate are positive. On X-axis on the right side of O, there is a unique point M corresponding to 2. On Y-axis, in the upper half-plane there will be a unique point N corresponding to 3. Draw lines from M and N, perpendicular to X-axis and to Y-axis respectively. The unique point P of their intersection is the point in the plane corresponding to (2, 3).

Now let us represent graphically the point corresponding to the ordered pair (−2, −3) in the plane. Both the coordinates of (−2, −3) are negative. On X-axis, on the left hand side of O, there is a unique point A corresponding to −2 and on Y-axis, in the lower half plane of the X-axis, there is a unique point B corresponding to −3. Draw lines perpendicular to X-axis from A and to Y-axis from B respectively. Their point of intersection, the unique point Q, is the point in the plane corresponding to (−2, −3). Similarly (−2, 3) and (2, −3) are represented as points R and S respectively (See figure 4.8).

We have seen that, a point x-coordinate of which is zero lies on the Y-axis and a point y-coordinate of which is zero lies on the X-axis. T represents (1, 0) and F represents (0, 2)

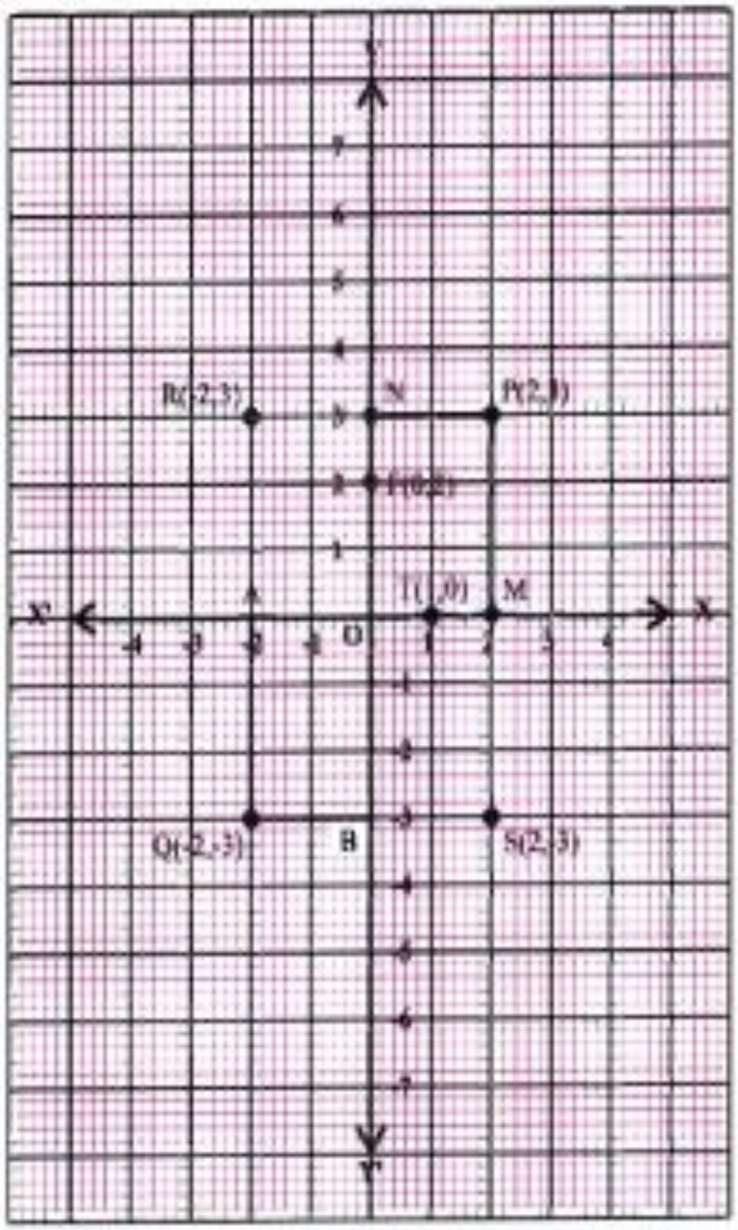


Figure.

From this illustration, we can say that to each ordered pair of real numbers, a unique point of the plane is associated. **(i)**

We have also seen that corresponding to each point in the plane there is a unique ordered pair of real numbers. **(ii)**

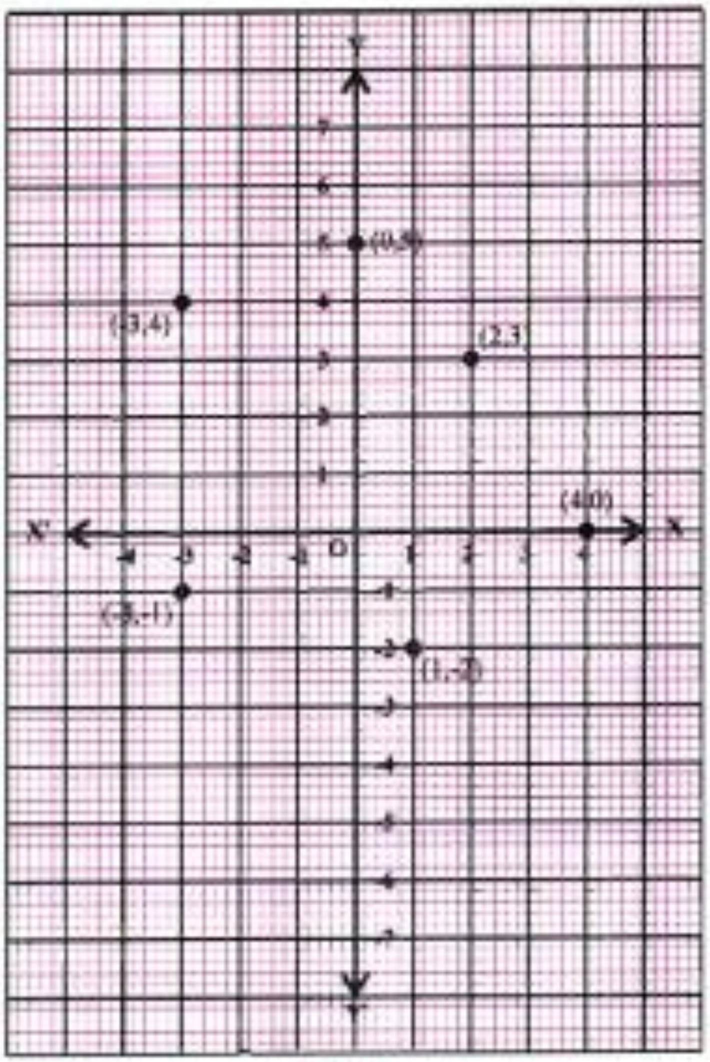
From (i) and (ii) we can say that there is a one-one correspondence between the plane and R × R and if a point P of the plane and the ordered pair (x, y) correspond to each other, then we write P (x, y).

P is called the representation of (x, y) in the plane and x and y are called cartesian co-ordinates of P. x is called the x-coordinate and y is called the y-coordinate of P. In fact, we identify P and (x, y) and say that (x, y) (like P) is a point of the plane.

By drawing graph of a set A × B we mean plotting of points of A × B in the Cartesian plane.

Example : Locate the point corresponding to ordered pairs.

$(-3, 4)$, $(-3, -1)$, $(4, 0)$, $(0, 5)$, $(1, -2)$ and $(2, 3)$ in the Cartesian plane.



Figure

Solution : Taking the scale 1 cm = 1 unit on the axes draw the X-axis and Y-axis on the graph paper. The positions of the points are shown by dots in the Fig. 4.9

Note : For the ordered pairs (a, b) and (p, q), $(a, b) = (p, q)$ if and only if $a = p$ and $b = q$. For example, let us find x and y, if $(5, 4y - 1) = (3x - 4, 7)$

Here, $(3x - 4, 7) = (5, 4y - 1)$

$\therefore$ $3x - 4 = 5$ and $4y - 1 = 7$

$\therefore$ $3x = 5 + 4$ and $4y = 7 + 1$

$\therefore$ $3x = 9$ and $4y = 8$

$\therefore$ $x = \frac{9}{3}$ and $y = \frac{8}{4}$

$\therefore$ $x = 3$ and $y = 2$

EXERCISE

1. Plot the following ordered pairs (x, y) in the plane :

   $(-4, -3)$, $(-3, 5)$, $(-2, -4)$, $(-1, 6)$, $(0, 2)$, $(1, -3.5)$, $(2, 3)$, $(4, -2)$.

2. Plot the points (x, y) in the cartesian plane obtained by taking values of x in the polynomial $y = 3x - 2$, $x = -3, -2, -1, 0, 1, 2, 3, 4$.

3. If P = $\{0, 1, -1\}$ and Q = $\{-3, 2\}$, then draw the graph of P × Q and Q × P.

4. If A = $\{-2, 3\}$ and B = $\{-1, 1, 4\}$, then draw the graphs of

   (1) A × B (2) B × A (3) A × A (4) B × B

5. Plot the points A(4, 5), B(−2, −1), C(−3, 6) and D(5, −2). From the graph, find the midpoints of $\overline{AB}$ and $\overline{CD}$.

6. Represent the points M(3, 4), N(−3, −2), P(−2, 5) and Q(4, −1) in the plane. Draw $\overleftrightarrow{MN}$ and $\overleftrightarrow{PQ}$. From the graph, find their point of intersection.

7. Examine the validity of the following statements :

   (1) Point (4, 0) lies on the X-axis.

   (2) P(−2, 3) is a point in the third quadrant.

   (3) For the point A, if the abscissa is 4 and the ordinate is −3, then A lies in the fourth quadrant.

   (4) The point of intersection of the axes has co-ordinates (0, 0).

   (5) In the plane the position of (y, x) is the same as the position of (x, y), where $x \neq y$.

   (6) B(0, −9) is a point on $\overrightarrow{OY}$.

   (7) For $x = 3$, $y = 2$, $u = -7$, $v = 11$ the point $(x - u, y - v)$ lies in the 1st quadrant.

   (8) Point (4, −5) lies in the lower half-plane of the X-axis and to the right hand side of Y-axis.

Paragraph -9

STATISTICS

Introduction

Everyday we come across a lot of information in the form of facts, numerical figures, tables, graphs etc. They are provided by newspapers, television media, magazines and other means of communications. These may relate to a batsman's average in cricket or bowling averages, profit-loss account of a company, temperatures of cities, expenditures in various sectors of a five year plan; percentage polling and so on. **These facts or figures, which are numerical or otherwise, collected with a certain purpose are called data.** Data is the plural form of the Latin word "datum".

The solutions to the problems pertaining to the basic sciences, sociology, agriculture, industry, management, administration etc. are sought today with the help of statistics. Though statistics is an old subject, it has become more prevalent from the beginning of the 20th century. When the administrators of any firm or department began to realise difficulties to bring about the solution to the problems, then the help from mathematicians and statisticians was sought. They collected data regarding the problems, analysed the collected data regarding the problems, scientifically evaluated the situation by constructing new principles based on mathematics and derived conclusions. When these conclusions proved to be very effective, the principles of statistics became very popular and progressive. **Thus statistics is a science dealing with the scientific methods of collecting, arranging, reducing, analysing the data and drawing proper and correct conclusions with the help of scientific principles.**

We have noticed that the base of statistics is data. For the solution of some problems or for certain predictions, the basic and important thing in statistics is data. In this chapter, we shall learn about data and other details regarding it.

Collection of Data

Let us start to collect data by the following activity.

Activity : We divide the students of our class into five groups. Assign each group the task to collect the data for one of the following information :

(i) Weight of 30 students of our class.

(ii) Number of family members in the families of 20 students of this class.

(iii) Height of 25 plants in or around our school.

(iv) Height of 20 students of our class.

(v) Total income of the family of 20 students of our class.

Now let us observe the results the students have collected.

How do they collect the data in each group ?

(i) Did they get the information from each and every student, house to house or personally contacted the head of the family for obtaining the information ?

(ii) Did they get the information from some source like school record available ?

For activities (i) to (iv) when the information is collected by the investigator himself or herself with a definite objective in his or her mind, the data obtained is called a **primary data.**

In activity (v), when the information was gathered from a source which is already stored in the school, the data obtained is called a **secondary data. Such data which has been collected by someone else in another context needs to be used with great care ensuring that the source is reliable.**

If the observations of the given data are expressed numerically, then it is said to be a **quantitative data** and if they are expressed non-numerically in qualitative form, then it is said to be a **qualitative data**. For example heights and weights of n students is a quantitative data, whereas the set of n observations obtained by tossing a balance coin n times is called a qualitative data.

EXERCISE

1. Classify the following data as primary data or secondary data :

(1) Number of students in the class.

(2) Election results obtained from print media or television news channels.

(3) Literacy rate figures obtained from educational survey.

(4) Number of trees in the school campus.

Presentation of Data

As soon as the work related to collect the data is over, the investigator has to find out ways to represent them in the form which is meaningful, easily understood and gives its main features at a glance. Sometimes the data available from sample survey is so large and extensive that it is difficult to derive conclusion from it, if it is not reduced or classified properly.

Let us find various ways of representing the data through illustrations

Range : The difference between the largest observation and the smallest observation is called range of the quantitative data.

As for example, consider the runs scored by Yusuf Pathan in 10 innnings as given : 37, 52, 25, 18, 22, 30, 54, 11, 41, 47.

The data in this form is called a raw data.

From the above data we can find the highest and the lowest number of runs. It is less time consuming if these were arranged in ascending or descending order. Let us arrange these numbers in ascending order as 11, 18, 22, 25, 30, 37, 41, 47, 52, 54

Now we can clearly see that the lowest score is 11 and highest score is 54.

$\therefore$ The range of this data is $54 - 11 = 43$.

When the number of observations in an experiment is large, the presentation of data in ascending or descending order is quite time consuming.

Moreover range does not give a clear picture of data. For example in above illustration the range is 43. But 43 is also the range in the following examples.

(i) 1, 44

(ii) 1001, 1044

(iii) 1, 2, 3, 4, 5, ..., 44

If the data is large, instead of arranging them in increasing or decreasing order, we prepare a table as follows.

The marks obtained by 30 students out of 100 students of class IX are as follows :

15	85	50	30	80	50	35	70	55	90
75	60	99	70	40	70	35	60	50	40
60	55	35	85	60	40	70	90	40	90

The number of students who have obtained certain number of marks is called the **frequency** of those marks. For example, 2 students got 85 marks. So the frequency of observation 85 is 2. To make the data more easily understandable, we write it in a table, as given below :

Table

Marks	15	30	35	40	50	55	60	70	75	80	85	90	99	Total
No. of students (i.e. the frequency)	1	1	3	4	3	2	4	4	1	1	2	3	1	30

Table is called an **frequency distribution table for ungrouped data** or simply a **frequency distribution table.**

Still an easier approach to prepare a table is to use telly marks. When an observation comes for the first time, we mark | against the class. For the observation occuring second time, we put || against the class in which it occurs. For a group of five observations symbol ~~||||~~ is used. For six observations we write ~~||||~~ | against the class containing the observation and so on.

The marks (out of 30) by 60 students of class IX in mathematics are as follows :

6	22	17	9	24	13	17	13	15	18	13	2	21	27	30
15	1	3	10	24	29	6	6	25	28	26	10	4	22	26
19	14	26	18	25	21	7	15	25	18	6	4	9	11	12
14	18	20	17	10	1	21	19	25	15	7	5	12	23	21

For such a large amount of data, we convert it into groups like 1 – 5, 6 – 10, 11 – 15, ..., 26 – 30 (since our data is from 1 to 30). These groups are called **classes** or **class intervals.**

The size of classes is called **class-size** or **class width** or **class length**, which is 5 here. In each of these classes the least possible observation of the class is called **lower class limit** of the class and the largest possible observation of the class is called the **upper class limit.**

Upper class limit of class 1-5 is 5.

Upper class limit of class 21-25 is 25 etc.

Lower class limit of class 6-10 is 6.

Lower class limit of class 16-20 is 16 etc.

Paragraph-10

LOGARITHM

Introduction

Previously we have learnt about powers and exponents. Also we have learnt about the properties of exponents.

For, $a, b \in R^+$, $x, y \in R$

(i) $a^x \cdot a^y = a^{x+y}$ (ii) $\frac{a^x}{a^y} = a^{x-y}$

(iii) $(a^x)^y = a^{xy}$ (iv) $(ab)^x = a^x \cdot b^x$ (v) $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

Logarithm

John Napier was born in 1550. He died on 4th April, 1667 in Edinburgh. A mathematician ***John Napier*** introduced the concept of logarithm for the first time in 17th century. Later, ***Henry Briggs***, a British mathematician born in Feb. 1561 in Yorkshire – England, prepared and published logarithm tables. He died on 26th January, 1663 in Oxford – England. Logarithm tables made complicated numerical calculations both – easy and fast. Today with the advent of desk calculators and computers, the work of numerical calculations has become easier and faster, thus reducing the usefulness of logarithm tables. All the while they are useful for calculations in the study of science and mathematics.

Definition : Let $a \in R^+ - \{1\}$ $y \in R^+$, $x \in R$ and let $a^x = y$. Then the value of x is called logarithm of y to the base a. It is denoted by $\log_a y$ (read as log y to the base a).

$\therefore$ $a^x = y$ if and only if $x = \log_a y$

From the above definition we can conclude that,

(i) we can obtain the logarithm of only positive real numbers.

(ii) for any $a \in R^+ - \{1\}$, $\log_a 1 = 0$, since $a^0 = 1$.

(iii) for every $a \in R^+ - \{1\}$, $\log_a a = 1$, since $a^1 = a$

(iv) for every $x \in R^+$, $y \in R^+$, $\log_a x = \log_a y$ if and only if $x = y$.

Properties of Logarithm

We will assume following properties of logarithm :

(1) If $a \in R^+ - \{1\}$, then $a^{\log_a x} = x$ ($x \in R^+$) and $\log_a a^x = x$ ($x \in R$).

Theorem 1 : Product rule

Let $a \in R^+ - \{1\}$.

Then for $x, y \in R^+$, $\log_a (xy) = \log_a x + \log_a y$

Corollary : If $x_1, x_2, x_3, ..., x_n \in R^+$ and $a \in R^+ - \{1\}$, then

$\log_a (x_1 x_2 x_3 ... x_n) = \log_a x_1 + \log_a x_2 + ... + \log_a x_n$

Theorem 2 : Quotient Rule

If $a \in R^+ - \{1\}$, and $x, y \in R^+$, $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

Corollary : $\log_a \left(\frac{1}{y}\right) = -\log_a y$; $a \in R^+ - \{1\}, y \in R^+$

Theorem 3 : Rule for the logarithm of a power

If $a \in R^+ - \{1\}, x \in R^+, n \in R$, then $\log_a x^n = n \log_a x$.

Example 1 : Simplify

(i) $\log_3 \left(\frac{17}{25}\right) + \log_3 \left(\frac{600}{119}\right) - \log_3 \left(\frac{8}{7}\right)$ (ii) $4\log_a \left(\frac{2}{7}\right) - 3\log_a \left(\frac{3}{49}\right) - \log_a \left(\frac{14}{9}\right)$

(iii) $\log_2 \left(\frac{\sqrt[3]{16}}{4}\right) + \log_3 \left(\frac{\sqrt{27}}{81}\right)$

Solution : (i) $\log_3 \left(\frac{17}{25}\right) + \log_3 \left(\frac{600}{119}\right) - \log_3 \left(\frac{8}{7}\right)$

$= \log_3 \left(\frac{17}{25} \times \frac{600}{119}\right) - \log \left(\frac{8}{7}\right)$

$= \log_3 \left(\frac{17}{25} \times \frac{600}{119} \div \frac{8}{7}\right)$

$= \log_3 \left(\frac{17}{25} \times \frac{600}{119} \times \frac{7}{8}\right)$

$= \log_3 3 = 1$

Paragraph-11

Important Points for Revision

- **A set is a well-defined collection of objects.**
- **A set without any member (element) is called a null set or an empty set.**
- **A set having only one member is called a singleton.**
- **$\in$ (belongs to) is an undefined symbol.**
- **If x is a member of the set A, we write $x \in A$**
- **If x is not a member of the set A, we write $x \notin A$.**
- **A set total number of members of which is a positive integer is called a finite set and a set which is not finite is called an infinite set. Null set is considered to be a finite set.**
- **If all the elements of a set A are present in the set B, then the set A is called a subset of the set B. This fact is denoted by $A \subset B$.**

Important points about subsets :

(1) **Empty set is a subset of every set. Thus, for any set A, $\emptyset \subset A$.**

(2) **Every set is a subset of itself. Thus, for any set A, $A \subset A$.**

(3) **If a set A has n elements, then number of its subsets is 2^n.**

(4) **$N \subset Z \subset Q \subset R$.**

- Generally, while dealing with a problem, we consider some definite set and its subsets. Such a definite set is called the **universal set** with reference to that problem. The **universal set** is denoted as U.

  A set which is a universal set for one problem need not be a universal set for another problem. For example, In Geometry, space or plane is a universal set. For interrelations of integers, set of integers Z is a universal set. For the solution of linear equations, the set of real numbers is a universal set.

- The set of all the elements which are in U but not in the given set A is called the **Complement of the set A**. It is denoted by A'.

  Thus, $A' = \{x \mid x \in U, x \notin A\}$

  so from the above definition, we get the following results.

  (1) $A \cup A' = U$ and (2) $A \cap A' = \emptyset$

- If two sets have same elements, they are said to be **equal sets**. **If every member of set A is a member of set B and every member of set B is a member of set A, then set A and set B are called equal sets. If A and B are equal sets we write A = B.** For equal sets A and B, $A \subset B$ and $B \subset A$.

i.e. if A $\subset$ B and B $\subset$ A, then A = B.

For example let $A = \{x \mid x \in N, x < 5\}$ and $B = \{1, 2, 3, 4\}$ be two sets.
Then both the sets A and B have the same members $\{1, 2, 3, 4\}$.
So, we say that A = B

- If every member of set A corresponds to one and only one member of set B and every member of set B corresponds to one and only one member of set A then the sets A and B are said to be in one-one correspondence with each other and the sets A and B are called **equivalent sets**. If set A is equivalent to set B, we write $A \sim B$.
- Thus, if two finite sets are in one-one correspondence with each other, then they should have the same number of elements.
- **Equal sets are always equivalent sets but equivalent sets need not be equal sets.**

For example, if $A = \{1, 2, 3\}$, $B = \{a, b, c\}$ then $A \sim B$ but $A \neq B$.
But for infinite sets, situation is different.

If, $E = \{2, 4, 6, 8, ...\}$, then $N \sim E$. Because for every element of N, A unique number n is related to the number $2n$ belonging E and for every element of E, a unique number m is related to the number $\frac{m}{2} \in N$. But $E \subset N$.

EXERCISE

1. Classify the following sets in (a) as empty set or singleton set and in (b) as equal sets or equivalent sets :

(a) (1) $A = \{x \mid x \in Z, x + 1 = 0\}$

(2) $B = \{x \mid x \in N, x^2 - 1 = 0\}$

(3) $C = \{x \mid x \in N, x$ is a prime number between 13 and 17$\}$

(b) (1) $A = \{x \mid x \in N, x \leq 7\}$,

$B = \{x \mid x \in Z, -3 \leq x \leq 3\}$

(2) $A = \{x \mid x \in N, x$ is a multiple of 2, $x < 10\}$,

$B = \{x \mid x \in N, x$ is an even natural number with a single digit$\}$

2. Find the number of subsets of the set $A = \{1, 2, 3\}$. Also write all the subsets of the set A.

3. If $A = \{x \mid x \in Z, x^2 - x = 0\}$, $B = \{x \mid x \in N, 1 \leq x \leq 4\}$, then can we say that $A \subset B$? Why ?

4. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 4, 6, 8\}$, then find A' and also verify that $A \cup A' = U$.

Paragraph-12

1. A number r is called a rational number, if it can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

2. A number s is called a irrational number, if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

3. The decimal expansion of a rational number is either terminating or non-terminating recurring. Moreover, a number whose decimal expansion is terminating or non-terminating recurring is rational.

4. The decimal expansion of an irrational number is non-terminating non-recurring. Moreover, a number whose decimal expansion is non-terminating non-recurring is irrational.

5. All the rational and irrational numbers make up the collection of real numbers.

6. There is a unique real number corresponding to every point on the number line. Also, corresponding to each real number, there is a unique point on the number line.

7. If r is rational and s is irrational, then $r + s$ and $r - s$ are irrational numbers, and rs and $\frac{r}{s}$ are irrational numbers, $r \neq 0$.

8. For positive real numbers a and b, the following identities hold:

   (i) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

   (iii) $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = a - b$ (iv) $\left(a + \sqrt{b}\right)\left(a - \sqrt{b}\right) = a^2 - b$

   (v) $\left(\sqrt{a} + \sqrt{b}\right)^2 = a + 2\sqrt{ab} + b$

9. To rationalise the denominator of $\frac{1}{\sqrt{a} + b}$, we multiply this by $\frac{\sqrt{a} - b}{\sqrt{a} - b}$, where a and b are integers.

10. Let $a > 0$ be a real number and p and q be rational numbers. Then

    (i) $a^p \cdot a^q = a^{p+q}$ (ii) $(a^p)^q = a^{pq}$

    (iii) $\frac{a^p}{a^q} = a^{p-q}$ (iv) $a^p b^p = (ab)^p$

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Paragraph-13

1. Sum of the angles of a quadrilateral is 360°.
2. A diagonal of a parallelogram divides it into two congruent triangles.
3. In a parallelogram,
   (i) opposite sides are equal (ii) opposite angles are equal
   (iii) diagonals bisect each other
4. A quadrilateral is a parallelogram, if
   (i) opposite sides are equal or (ii) opposite angles are equal
   or (iii) diagonals bisect each other
   or (iv) a pair of opposite sides is equal and parallel
5. Diagonals of a rectangle bisect each other and are equal and vice-versa.
6. Diagonals of a rhombus bisect each other at right angles and vice-versa.
7. Diagonals of a square bisect each other at right angles and are equal, and vice-versa.
8. The line-segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it.
9. A line through the mid-point of a side of a triangle parallel to another side bisects the third side.
10. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral, in order, is a parallelogram.

Paragraph-14

Circles and Its Related Terms: A Review

Take a compass and fix a pencil in it. Put its pointed leg on a point on a sheet of a paper. Open the other leg to some distance. Keeping the pointed leg on the same point, rotate the other leg through one revolution. What is the closed figure traced by the pencil on paper? As you know, it is a circle (see Fig.10.2). How did you get a circle? You kept one point fixed (A in Fig.10.2) and drew all the points that were at a fixed distance from A. This gives us the following definition:

Fig. 10.2

The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.

The fixed point is called the *centre* of the circle and the fixed distance is called the *radius* of the circle. In Fig.10.3, O is the centre and the length OP is the radius of the circle.

Fig. 10.3

Remark : Note that the line segment joining the centre and any point on the circle is also called a *radius* of the circle. That is, 'radius' is used in two senses-in the sense of a line segment and also in the sense of its length.

You are already familiar with some of the following concepts from Class VI. We are just recalling them.

A circle divides the plane on which it lies into three parts. They are: (i) inside the circle, which is also called the *interior* of the circle; (ii) the *circle* and (iii) outside the circle, which is also called the *exterior* of the circle (see Fig.10.4). The circle and its interior make up the *circular region*.

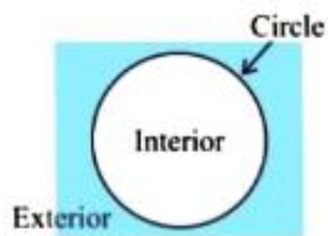

Fig. 10.4

If you take two points P and Q on a circle, then the line segment PQ is called a *chord* of the circle (see Fig. 10.5). The chord, which passes through the centre of the circle, is called a *diameter* of the circle. As in the case of radius, the word 'diameter' is also used in two senses, that is, as a line segment and also as its length. Do you find any other chord of the circle longer than a diameter? No, you see that a *diameter is the longest chord and all diameters have the same length, which is equal to two* *times the radius*. In Fig.10.5, AOB is a diameter of the circle. How many diameters does a circle have? Draw a circle and see how many diameters you can find.

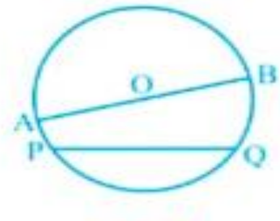

Fig. 10.5

A piece of a circle between two points is called an *arc*. Look at the pieces of the circle between two points P and Q in Fig.10.6. You find that there are two pieces, one longer and the other smaller (see Fig.10.7). The longer one is called the *major arc* PQ and the shorter one is called the *minor arc* PQ. The minor arc PQ is also denoted by $\overparen{PQ}$ and the major arc PQ by $\overparen{PRQ}$, where R is some point on the arc between P and Q. Unless otherwise stated, arc PQ or $\overparen{PQ}$ stands for minor arc PQ. When P and Q are ends of a diameter, then both arcs are equal and each is called a *semicircle*.

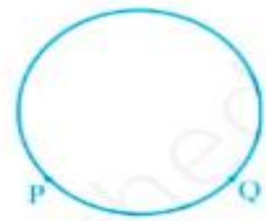

Fig. 10.6

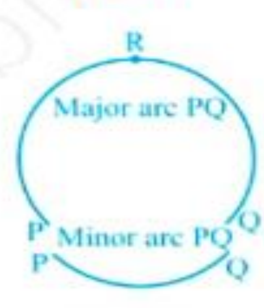

Fig. 10.7

The length of the complete circle is called its *circumference*. The region between a chord and either of its arcs is called a *segment* of the circular region or simply a *segment* of the circle. You will find that there are two types of segments also, which are the *major segment* and the *minor segment* (see Fig. 10.8). The region between an arc and the two radii, joining the centre to the end points of the arc is called a *sector*. Like segments, you find that the minor arc corresponds to the *minor sector* and the major arc corresponds to the *major sector*. In Fig. 10.9, the region OPQ is the minor sector and remaining part of the circular region is the major sector. When two arcs are equal, that is, each is a semicircle, then both segments and both sectors become the same and each is known as a *semicircular region*.

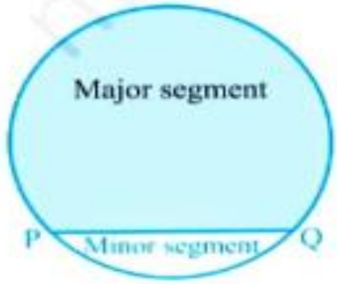

Fig. 10.8

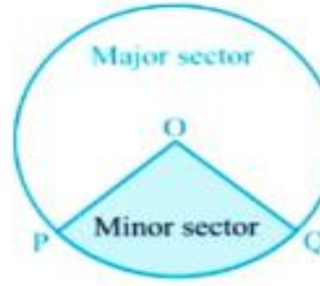

Fig. 10.9

Paragraph-15

1. Surface area of a cuboid = $2(lb + bh + hl)$
2. Surface area of a cube = $6a^2$
3. Curved surface area of a cylinder = $2\pi rh$
4. Total surface area of a cylinder = $2\pi r(r + h)$
5. Curved surface area of a cone = πrl
6. Total surface area of a right circular cone = $\pi rl + \pi r^2$, i.e., $\pi r(l + r)$
7. Surface area of a sphere of radius $r = 4\pi r^2$
8. Curved surface area of a hemisphere = $2\pi r^2$
9. Total surface area of a hemisphere = $3\pi r^2$
10. Volume of a cuboid = $l \times b \times h$
11. Volume of a cube = a^3
12. Volume of a cylinder = $\pi r^2 h$
13. Volume of a cone = $\frac{1}{3}\pi r^2 h$
14. Volume of a sphere of radius $r = \frac{4}{3}\pi r^3$
15. Volume of a hemisphere = $\frac{2}{3}\pi r^3$

[Here, letters l, b, h, a, r, etc. have been used in their usual meaning, depending on the context.]

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